Network Analysis – PERT, CPM 11

SLOB Mapped against the Module

To equip oneself with application-oriented knowledge of analysing Network to facilitate management decisions for optimisation through resource allocation, managing competition, work scheduling and managing cost overrun, demand estimation, production and cost analysis etc.

Module Learning Objectives

After studying this module, the students will be able to:

- Develop simple Network diagrams with Activities and Events.
- Identify Critical Path through the calculation of the Earliest expected time and Latest allowable time.
- Compute the values of Slack of events and Float of activities.
- Examine the situations which suggest the use of each of CPM and PERT.
- Estimate the probability of completion of a project within a stipulated time period.
- Review in detail the concepts and use of PERT Cost to plan and control projects.
- Show the use of Crashing in the planning and control of projects.

Network Analysis – PERT, CPM

11

Network analysis is the general name given to certain specific techniques which can be used for planning, management and control of project. It often acts as a network management tool for breaking down projects into components or individual activities and recording the result on a flow chart or network diagram. These results generally reveal information that is used to determine duration, resource limitations and cost estimates associated with the project.

It offers insight into what is occurring at each critical point of the network. Project management and efficient resource allocation are two critical aspects of the production and operations managers' responsibilities. Since a project is non-repetitive and temporal in nature, the mode of management differs from the usual job shop or other related types of scheduling.

Network analysis enables us to take a systematic quantitative structural approach to the problem of managing a project through to successful completion. Since it has a graphical representation, it can be easily understood and used also by those with less technical background.

Network is a graphical representation of all the Activities and Events arranged in a logical and sequential order. Network analysis plays an important role in project management. A project is a combination of interrelated activities all of which must be executed in a certain order for its completion. Activity is the actual performance of the job. This consumes resources (Time, human resources, money, and material. An event refers to start or completion of a job. This does not consume any resources.

CPM and PERT

CPM (Critical Path Method) is a project modeling technique developed in the late 1950s by Morgan R. Walker of Du Pont and James E. Kelly, Jr. of Remington Rand. Test of the technique was done in 1958 when it was applied for the first time in construction of a Chemical Plant. Subsequently in March 1959 it was applied for Planning, Scheduling and Controlling the activities related to Shutdown Maintenance of Du Pont's Plant in Louisville, Kentucky and a sizeable amount of reduction in non-productive working hours was observed due to its presence. CPM is also known as CPA or Critical Path Analysis. It is commonly used with all forms of projects including Construction, Aerospace and Defense, Software Development, Research Projects, Product Development, Plant Maintenance etc. In any project with interdependent activities this technique of mathematical analysis can be successfully applied.

PERT (Program Evaluation and Review Technique) was first devised in 1958 for the Polaris Submarine Missile Program of U.S Navy. The Navy's special Projects office, entrusted with developing the Polaris Submarine Weapon system and the Fleet Ballistic Missile system, developed a statistical technique for measuring and forecasting progress in research and development programs. This technique is applied as a decision making tool designed to save time in achieving end objectives. It is of particular interest to those who are engaged in research and development programs for which time is a critical factor. This technique takes recognition of three factors which influence successful achievement of R & D Program objectives – time, resource and technical performance specifications.

Both CPM and PERT are essentially Network oriented techniques using the same principle. Both are basically time oriented methods and lead to determination of time schedule for a project. The significant difference between two approaches is the fact that the time estimates for the different activities in CPM are assumed to be deterministic, while in PERT they are considered as probabilistic. These techniques are referred as Project Scheduling techniques.

Difference between PERT and CPM

PERT	CPM
1. It is a technique for planning scheduling & controlling of projects whose activities are subject to uncertainty in the performance time. Hence it is a probabilistic model.	1. It is a technique for planning scheduling & controlling of projects whose activities are not subjected to any uncertainty and the performance times are fixed. Hence it is a deterministic model.
2. It is an Event oriented system	2. It is an Activity oriented system
3. Basically does not differentiate critical and non-critical activities.	3. Differentiates clearly the critical activities from the other activities.
4. Used in projects where resources (men, materials, money) are always available when required.	4. Used in projects where overall costs is primarily important. Therefore better utilization of resources is noticed in it.
5. Suitable for Research and Development projects where times cannot be predicted.	5. Suitable for civil constructions.

Applications

(i) Construction of a Residential complex, Commercial Complex, Bridges, Factories, Highways etc.

(ii) Installation of complex Equipment in a factory

(iii) Maintenance and overhauling complicated equipment in Petro-chemical complex, Power plants, Steel plants etc.

(iv) Ship building, Aircraft building etc.

(v) Satellite and Missile development

(vi) Installation of a pipe line project.

(vii) Organizing big events like Olympic, Soccer World Cup, Big conferences etc.

(viii) Shifting of manufacturing plant from one location to other.

Basic Terminology related to Network

Activity – As mentioned before, all projects may be viewed as being composed of operations or tasks called Activities which require the expenditure of time and resources for their accomplishments. An Activity is depicted by a single arrow on the project network. The activity arrow is not scaled; the length of the activity arrow is only a matter of convenience and clarity and does not represent importance of time. The head of the arrow shows the sequence or flow of activities. An activity cannot begin until the completion of the preceding activity or activities. It is important that activities be defined beforehand so that the beginning and end of each activity can be identified clearly.

Predecessor Activity – Activity or Activities which must be completed immediately prior to the start of another activity or activities are called Predecessor Activity.

Successor Activity – Activities that cannot be started until one or more of the other activities are completed, but immediately succeed them are called Successor Activities.

Concurrent Activity – Activities which can be accomplished simultaneously are known as Concurrent Activities. It can be mentioned that an activity can be a predecessor or successor to an event or it may be concurrent with one or more of the other activities.

Event – An Event represents a specific accomplishment in the project and takes place at a particular instant of time. Therefore it does not consume time or resources. An Event in a Network is a time oriented reference point that signifies the end of one activity and the beginning of another. Events are usually represented in the Network diagram as circles. *The event circles are known as Nodes.* All activity arrows must begin and end with event nodes as shown below. Here A is the Start Event and B is the Finish Event for the Activity (A – B).

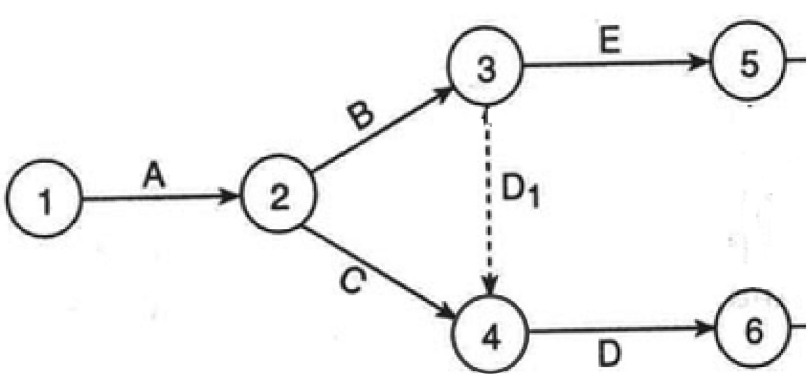
Merge Event – The Event where more than one activity ends is called the Merge Event. In the diagram below, Event 4 is a Merge Event because both the Activities C and D_1 finish here.

Burst Event – The Event from where more than one activity starts is called the Burst Event. In the diagram below Event 2 is a Burst Event because both the Activities B and C start from here.

Merge and Burst Event – The Event where more than one activity finishes and also from where more than one activity starts is called Merge and Burst Event.

Dummy Activity – Sometimes in project network it is possible that two concurrent activities have same start and end events. To avoid such situation and make activities distinguishable Dummy Activity is introduced. As a result, each activity can be identified by unique end event. **Dummy Activities consume no time or resources**. As a convention Dummy Activities are represented by dashed arrows in the Network diagram. It is inserted in the network to clarify activity pattern in the following situations –

1. To make activities with common starting and finishing events distinguishable.
2. To identify and maintain the proper precedence relationship between activities those are not connected by events.

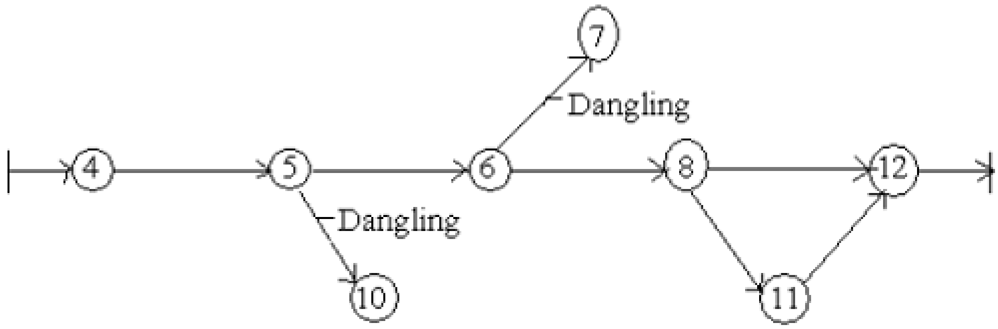


In the diagram above, Activities B and C are concurrent, E is dependent on B and D is dependent on both B and C. Such a situation is handled by Dummy Activity D_1.

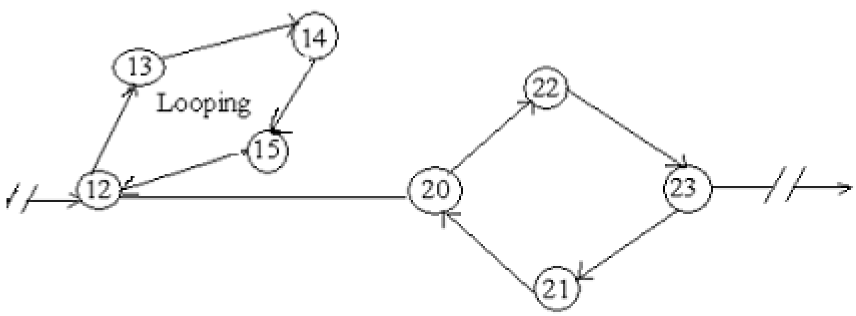
Common Errors in drawing Network diagrams

Three types of errors are most commonly observed in drawing Network diagrams. They are –

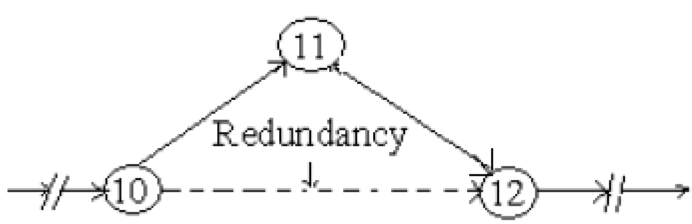
1. **Dangling** – To disconnect an activity before the completion of all activities in a Network diagram is known as Dangling. In the figure below, activities (5 – 10) and (6 – 7) are not the last activities in the Network. So the diagram is incorrect and has the error of Dangling. This is seen in both AOA and AON diagrams.



2. **Looping or Cycling** – Drawing an endless loop in a Network is called Looping or Cycling error. It is shown below. This is seen in both AOA and AON diagrams.



3. **Redundancy** – Unnecessary insertion of Dummy Activity in a Network diagram is known as the error of Redundancy. It is shown in the diagram below. This type of error is seen only in AOA diagrams.



[**Note:** There can be two types of Network diagrams – AOA (or Activity on Arrow) and AON (or Activity on Node) diagrams. As the name implies, AOA diagrams have activities represented by arrows and AON diagrams have activities represented by nodes or circles. Though arrows are used in AON diagrams also, but they are meant only for representing the logical relationships going from predecessor to successor].

Rules for drawing Network diagram

In a network diagram, arrows represent the activities and circles represent the events.

(i) The tail of an arrow represents the start of an activity and the head represent the completion of the activity. Event at the tail end of the activity is called Tail Event and the one at the head end is called Head Event.

(ii) Head events always have number higher than the Tail events. Thus for the Activity (i – j) it goes without saying i < j always.

(iii) Time flows from left to right of the Network. Thus no Activity of the Network should have arrowhead pointing towards left.

(iv) Activity arrows should not be crossed unless it is completely unavoidable.

(v) Activity arrows should be kept straight and not curved or bend.

(vi) The arrows depicting various activities are indicative of the logical precedence only. The length and bearing of the arrows are of no significance.

(vii) The event number 1 denotes the start of the project and is called initial event.

(viii) Event carrying the highest number in the network denotes the completion of the project and is called terminal event.

(ix) Each defined activity is represented by one and only one arrow in the network.

(x) No two activities can be identified by the same beginning and end events. In such cases, a dummy activity should be introduced to resolve the issue.

(xi) Determine which operation must be completed immediately before other can start.

(xii) Determine which other operation must follow the other given operation.

(xiii) The network should be developed on the basis of logical, analytical and technical dependencies between various activities of the project.

(xiv) Dangling must be avoided in a Network diagram. This happens when precedence and inter relationship of the activities are not properly identified. In fact this is an error.

(xv) Unnecessary Dummy Activities must not be drawn while preparing a Network diagram. This is an error known as Redundancy.

Procedure of drawing a Network Diagram

A project consists of tasks with definite starting and ultimate ending points and hence a project manager is saddled with the responsibilities of getting job done on schedule within allowable cost and time constraint specified by the management. Typically all projects can be broadly broken into:

(a) Separate activities – where each activity has a duration (time from the start of the activity to its finish).

(b) Precedence relationships – which govern order in which the activities are to be performed.

The main problem is to bring all these activities together in a coherent fashion to complete the project at the stipulated time.

With this background, the procedure of drawing a Network Diagram can be given as follows -

1. Specify the Individual Activities: From the work breakdown structure, a listing can be made of all the activities in the project. This listing can be used as the basis for adding sequence and duration information in later steps.
2. Determine the Sequence of the Activities: Some activities are dependent on the completion of others. A listing of the immediate predecessors of each activity is useful for constructing the CPM network diagram.
3. Draw the Network Diagram: Once the activities and their sequencing have been defined, the CPM diagram can be drawn. CPM originally was developed as an activity on node (AON) network, but some project planners prefer to specify the activities on the arrows.
4. Estimate Activity Completion Time: The time required to complete each activity can be estimated using past experience or the estimates of knowledgeable persons. CPM is a deterministic model that does not take into account variation in the completion time, so only one number is used for an activity's time estimate.
5. Identify the Critical Path: The critical path is the longest-duration path through the network. The significance of the critical path is that the activities that lie on it cannot be delayed without delaying the project. Because of its impact on the entire project, critical path analysis is an important aspect of project planning.

The critical path can be identified by determining the four parameters for each activity. The four parameters are Earliest Start, Earliest Finish, Latest Finish and Latest Start.

Time Estimates and Critical Path in Network Analysis

Time estimates are extremely important for planning various activities of a project. Activity time is a forecast of the time an activity is expected to take from its start to finish under normal conditions. In fact estimation of activity time is rather difficult and requires lot of hands on experience for a specific type of job. Thus an assumption is made that activity times are available in time units. These values are put in the Network diagram either on the top or the bottom of the Activity arrow.

The main objective of the time analysis is to get a planned schedule of the project which includes –

(i) Completion time for the project,

(ii) Earliest time when each activity can begin

(iii) Latest time when each activity can begin without delaying the project.

(iv) Float for each activity (meaning the amount of time by which the completion of an activity can be delayed without delaying the project completion)

(v) Identification of critical activities and critical path.

For further discussion on the above mentioned time estimates following notations are used.

(**i**, j) = Activity (**i, j**) with tail end number **i** and head end number **j**

E_i = Earliest occurrence time of event **i**

L_j = Latest allowable occurrence time of event **j**

t_{ij} = Time estimate of activity (**i, j**)

ES_{ij} = Earliest Starting time of activity (**i, j**)

EF_{ij} = Earliest Finish time of activity **(i, j)**

LS_{ij} = Lastest Starting time of activity **(i, j)**

LF_{ij} = Latest Finish time of activity **(i, j)**

Basic computations for scheduling are done using the following concepts.

(A) **Forward Pass** (for Earliest Event times) – Computation using this concept yields the Earliest Start and Earliest Finish timings for each activity as well as the Earliest expected occurrence time for each event.

- The computations begin from the START Node and move forward to complete at the END Node. To accomplish this, an assumption is made that the earliest occurrence time for the **Start Node** is **zero**.
- Earliest Starting time of activity **(i, j)** is the earliest event time of the Tail event, that is $ES_{ij} = E_i$
- Earliest Finishing time of activity **(i, j)** is the Earliest Starting time plus the activity duration, that is $EF_{ij} = ES_{ij} + t_{ij}$
- Earliest Event time for event **j** is the *Maximum* of the Earliest Finish times of all the activities ending into that event. Thus E_j = Maximum of all $(ES_{ij} + t_{ij})$ values

(B) **Backward Pass** (for Latest allowable Event times) – The Latest event time (L) means the time by which all activities merging into an event must be completed without delaying the total project. These are computed by reversing the method of calculation used for earliest event times.

- The computations begin from the END Node and move backward to complete at the START Node. Assumption in this case is **L = E** for the **End Node**.
- Latest Finish time of activity **(i, j)** is the Latest Event time for **j**, that is $LF_{ij} = L_j$
- Latest Starting time of activity **(i, j)** is the Latest Finish time of **(i, j)** minus the activity duration, that is $LS_{ij} = LF_{ij} - t_{ij}$
- Latest Event time for event **i** is the *Minimum* of the Latest Start time of all the activities originating from that event. Thus L_i = Minimum of all $(LF_{ij} – t_{ij})$ values

(C) **Slack of an Event** – It is the difference between the Latest and Earliest times of an Event. Hence it can be written that Slack of the Event **i** = $L_i - E_i$

For the activity **(i, j)** Head Event is j and Tail Event is i. If Slack is calculated for the Head Event then that is called **Head Slack** and for the Tail Event it is called **Tail Slack.** Thus for activity **(i, j)** Head Slack = $L_j – E_j$ and Tail Slack = $L_i - E_i$

An Event with **Zero Slack** is known as **Critical Event.**

(D) **Float of an Activity** – There can be three types of Floats for an Activity which are as follows –

Total Float – It is defined as the amount of time by which completion of an activity can be delayed beyond the earliest expected completion time without affecting the project duration. In other words the Total Float of an Activity (i, j) is the difference between the Latest Start and Earliest Start of that activity. Thus Total Float $\mathbf{(TF_{ij}) = LS_{ij} - ES_{ij} = (L_j – E_i) – t_{ij}}$

The value of Total Float for any Activity can help in making conclusion as follows –

Total Float < 0 or Negative Total Float indicates that the resources are not adequate which might cause delay in finishing the activity. Thus induction of extra resources becomes necessary to avoid delay in activity completion.

Total Float = 0 means **resources are just sufficient to complete the activity on time**. In other words, any slackness in arranging the resources for the activity will lead to delay in its completion.

Total Float > 0 or Positive Total Float indicates *that the resources are extra*. Thus one has the freedom to reallocate the resources.

An Activity with **Zero Total Float** is known as **Critical Activity.**

Free Float – This is concerned with commencement of subsequent activity. It is defined as the time by which an activity can be delayed beyond the earliest finish time without affecting the earliest start of a subsequent activity. For the activity **(i, j)** it is given by, Free Float (FF_{ij}) = $(E_j - E_i) - t_{ij}$

This can also be expressed as Free Float = $(E_j - E_i) - t_{ij} + L_j - L_j = [(L_j - E_i) - t_{ij}] - (L_j - E_j)$

Or, **Free Float = Total Float – Head Slack**

Independent Float – This is concerned with prior and subsequent activities. It is defined as the amount of time by which the start of an activity can be delayed without affecting the earliest start time of any immediately following activity, assuming that the preceding activity has finished at its latest time. For the activity (i, j) it is given by, Independent Float (IF_{ij}) = $(E_j - L_i) - t_{ij}$

This can also be expressed as **Independent Float = Free Float – Tail Slack**

(E) **Important points to note** – Following important points related to Float and Slack can be noted.

1. Independent Float ≤ Free Float ≤ Total Float
2. The knowledge of Floats facilitates to have an idea about the underutilized resources and flexibility of the schedule. This helps to know the extent to which the resources can be utilized on different activities.
3. Float can be used for redeployment of resources to level the same or to reduce project duration. But at the same time whenever the float in a particular activity is utilized, the float of not only that activity, but that of other activities will also change.
4. The basic difference between Slack and Float is the fact that Slack is used for Events and Float is used for Activities.

(F) **Critical Path** – A Path in a Network diagram refers to the sequence of activities such that it begins at the starting event and ends at the final event. The length or duration of a path is the sum of the durations of each individual activity lying on the path.

The sequence of critical activities in a Network diagram is called **Critical Path**. The Critical Path is the longest path or the path of longest duration in the network from the starting event to the ending event. In other words, it defines the minimum time required to complete the project.

If the activities on Critical Path are delayed, the project will also be delayed by the same amount of time unless the durations of the future critical activities are reduced. The critical path of a network is denoted by darker or double lines to distinguish it from the non-critical paths.

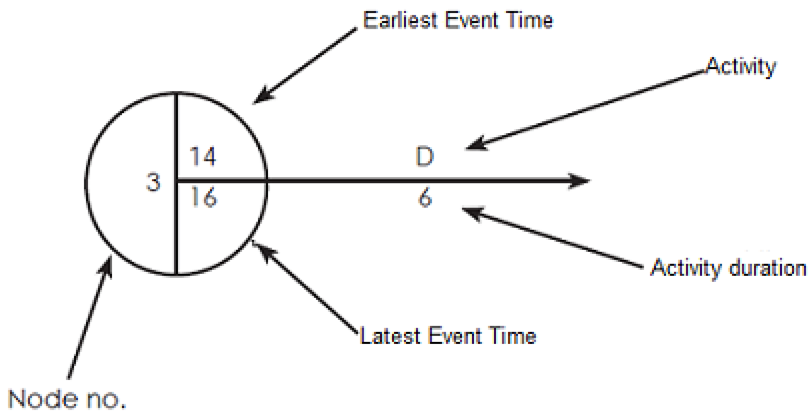
Some important features of the Critical Path are as follows –

1. If the duration of a project is to be shortened, duration of some of the activities on the critical path must be shortened. Induction of additional resources will not give the desired result unless the critical path is shortened first
2. The variation in actual performance time from the expected activity duration will be completely reflected in one-to-one fashion in the anticipated completion of the project.

(G) Representation of an Event in a Network diagram with Earliest and Latest times

It has been discussed before that Event or Node of a project is denoted by a circle in a Network diagram. In general the circle is divided into three parts – a semicircle and two quadrants for the purpose of its representation in the Network. The space of the semicircle is used for writing Event or Node Number and those of the two quadrants are used for writing the Earliest and the Latest Event times for the particular event

The diagram below shows how an Event is represented in a Network diagram –



- The Node number is 3
- The Earliest Event Time is 14 days which is also the Earliest Start Time (EST) of the succeeding activity D
- The Latest Event Time is 16 days which is also the Latest Finish Time for the preceding activity as well as the Latest Start Time (LST) of the succeeding activity D
- There is 2 days' float in this case (difference between EST and LST)
- The activity that follows the node is labeled as 'D' and it is of duration 6 days

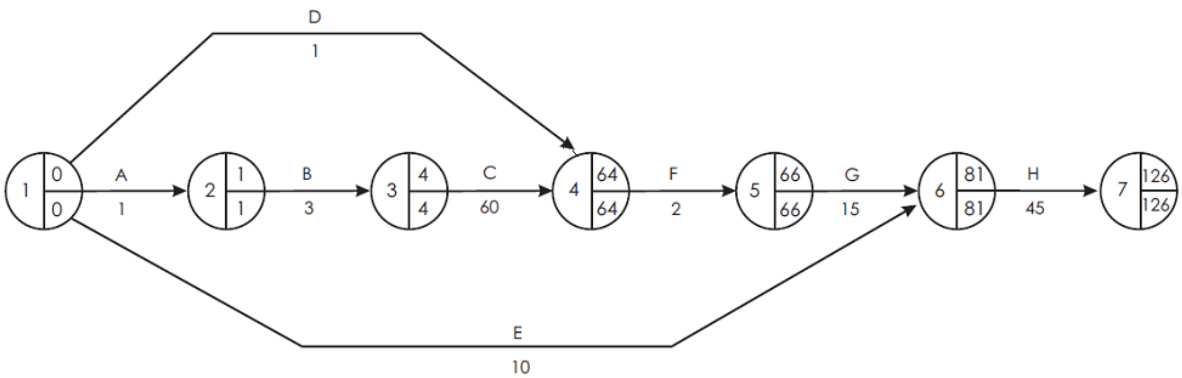
Illustration 1

The following table gives the activities and other relevant information related to "Making of a loaf".

Activity	Preceded by	Elapsed Time (Minutes)
A - Weigh ingredients	-	1
B - Mix ingredients	A	3
C - Dough rising time	B	60
D - Prepare tins	-	1
E - Pre-heat oven	-	10
F - Knock back dough and place in tins	C&D	2
G - 2nd dough rising time	F	15
H - Cooking time	E & G	45

Draw a Network diagram. Also find the Earliest and Latest Times of each Event of the Network. Identify the different paths of the Network and their corresponding durations. Which path is critical? Find the time required to complete the job.

Solution:



The Earliest expected Time and the Latest allowable Time of each event is determined by using the methods of Forward Pass and Backward Pass respectively.

The formula used in the calculation of Earliest expected Event Time is $E_j = E_i + t_{ij}$ & for Merge Events $E_j = \text{Max} (E_i + t_{ij})$

As per the standard procedure of Forward Pass $\mathbf{E_1} = 0$

$\mathbf{E_2} = E_1 + t_{12} = 0+1 = 1$, $\mathbf{E_3} = E_2 + t_{23} = 1+3 = 4$, $\mathbf{E_4} = \text{Max.} [(E_3 + t_{34}), (E_1 + t_{14})] = \text{Max.} [(4+60), (0+1)] = 64$

$\mathbf{E_5} = E_4 + t_{45} = 64+2 = 66$, $\mathbf{E_6} = \text{Max.} [(E_5 + t_{56}), (E_1 + t_{16})] = \text{Max.} [(66+15), (0+10)] = \text{Max.} [81, 10] = 81$

$\mathbf{E_7} = E_6 + t_{67} = 81+45 = 126$

These values are shown in the upper one of the two quadrants drawn in each circle to represent events.

The formula used in the calculation of Latest allowable Event Time is $L_i = L_j - t_{ij}$ & for Burst Events $L_i = \text{Min.} (L_j - t_{ij})$

As per the standard procedure of Backward Pass L = E for the last event. Thus $\mathbf{L_7} = E_7 = 126$

$\mathbf{L_6} = L_7 - t_{67} = 126 - 45 = 81$, $\mathbf{L_5} = L_6 - t_{56} = 81 - 15 = 66$, $\mathbf{L_4} = L_5 - t_{45} = 66 - 2 = 64$, $\mathbf{L_3} = L_4 - t_{34} = 64 - 60 = 4$,

$\mathbf{L_2} = L_3 - t_{23} = 4 - 3 = 1$, $\mathbf{L_1} = \text{Min.} [(L_2 - t_{12}), (L_4 - t_{14}) \ \& \ (L_6 - t_{16})] = \text{Min.} [(1\text{-}1), (64\text{-}1) \ \& \ (81\text{-}10)] = 0$

These values are shown in the lower one of the two quadrants drawn in each circle to represent events.

From the above diagram, different paths of the Network and their corresponding durations are as follows –

1. A – B – C – F – G – H or 1 – 2 – 3 – 4 – 5 – 6 -- 7 & Duration = 1+3+60+2+15+45 = 126 Minutes
2. D – F – G – H or 1 – 4 – 5 - 6 – 7 & Duration = 1+2+15+45 = 63 Minutes
3. E – H or 1 – 6 – 7 & Duration = 10+45 = 55 Minutes

Of the above three, the duration of the path A – B – C – F - G – H is maximum. So this is the CRITICAL PATH. Time required to make the loaf is 126 Minutes.(This is the project completion time)

[**Note:** In this example, there is a clear sequence of events that have to happen in the right order. If any of the events on the critical path is delayed, then the bread will not be ready as soon. However, tasks D (prepare tins) and E (heat the oven) can be started at any time as long as they are done by the latest time in the following node.

So, we can see that the oven could be switched on as early as time 0, but we can work out that it could be switched on at any time before 71 – any later than this and it won't be hot enough when the dough is ready for cooking. There is some 'float' available for tasks D and E as neither is on the critical path.]

Illustration 2

Activity	Preceded by	Elapsed Time (Minutes)
A - Weigh ingredients	-	1
B - Mix ingredients	A	3
C - Dough rising time	B	60
D - Prepare tins	-	1
E - Pre-heat oven	-	10
F - Knock back dough and place in tins	C&D	2
G - 2nd dough rising time	F	15
H - Cooking time	E & G	45

Using the above information of Illustration 1, compute Total, Free and Independent Floats for each Activity.

Solution:

Activity (i – j)	Duration (Minutes)	Earliest Time in Minutes		Latest Time in Minutes		*Slack in Minutes		Float in Minutes		
		Start	Finish	Start	Finish	Head	Tail	Total	Free	Independent
[1]	[2]	[3}	[4]*	[5]*	[6]	[7]	[8]	[9]*	[10]*	[11]*
A (1-2)	1	0	1	0	1	0	0	0	0	0
B (2-3)	3	1	4	1	4	0	0	0	0	0
C (3-4)	60	4	64	4	64	0	0	0	0	0
D (1-4)	1	0	1	63	64	0	0	63	63	63
E (1-6)	10	0	10	71	81	0	0	71	71	71
F (4-5)	2	64	66	64	66	0	0	0	0	0
G (5-6)	15	66	81	66	81	0	0	0	0	0
H (6-7)	45	81	126	81	126	0	0	0	0	0

Note: [4]* = [3] + [2], [5]* = [6] – [2], [9]* = [5]* - [3] or [6] – [4]* [10]* = [9]* - [7], [11]* = [10]* - [8]

*Slack is the difference between Earliest and Latest Times of an Event. Calculation of these times have already been shown in the previous Illustration. Also these values are given in the different Nodes of the diagram. Thus Head and Tail slacks are calculated using the figures in the diagram.

From above it is clear that the Activities D and E are having positive Floats which outright indicate that the activities are non-critical. Thus there is some cushion for both these activities and they can be delayed without affecting the ultimate completion time of the job i.e 126 Minutes.

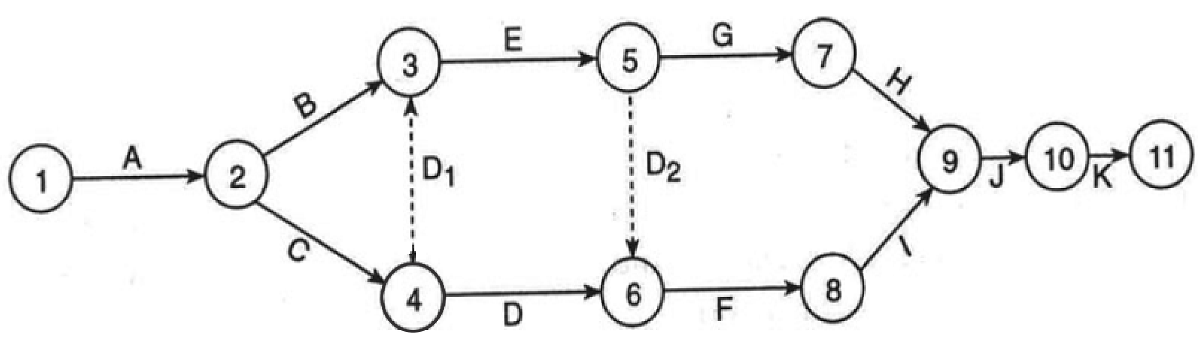
Illustration 3

XYZ Auto-manufacturing Co. has to prepare a design of its latest model of motorcycle. The various activities to be performed to prepare design are given in the following table: Prepare a Network diagram.

Activity	Description of activity	Preceding activity
A	Prepare drawing	—
B	Carry out cost analysis	A
C	Carry out financial analysis	A
D	Manufacture tools	C
E	Prepare bill of material	B, C
F	Receive material	D,E
G	Order sub-accessories	E
H	Receive sub-accessories	G
I	Manufacture components	F
J	Final assembly	I,H
K	Testing and shipment	J

Solution:

The Network Diagram with two Dummies D_1 & D_2 is as shown below:



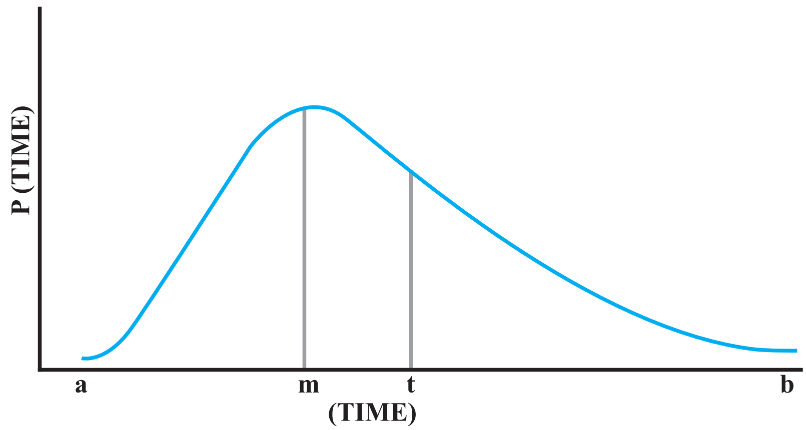
Three Time Estimates of PERT

In Critical Path Method or CPM, the Activity durations are assumed to be known with certainty. But for certain projects like R & D Projects or Projects related to New Product introduction etc. it is not possible to know the Activity durations with certainty due to involvement of various uncertainties. In such cases Program Evaluation and Review Technique or PERT is used where help of three time estimates are taken for each activity to subsequently get the Expected Time for each activity. These three Time Estimates are discussed below –

Optimistic Time – It is that time estimate of an activity when everything is assumed to go well as per plan. In other words it is the estimate of minimum possible time, which an activity takes for completion under ideal conditions. It is denoted as ‘a’ or t_o

Most Likely Time – It is the time which the activity will take most of the time if performed number of times. In other words it is the modal value. This is denoted by 'm' or t_m

Pessimistic Time – It is the unlikely but possible performance time if whatever could go wrong, goes wrong in series. In other words it is the maximum possible time which an activity can conceivably take under totally unfavorable conditions. It is denoted as 'b' or t_p



Above mentioned time estimates of an Activity follows Beta (β) Probability Distribution. It is assumed that the three time estimates form the end points and mode of Beta Distribution. It is further assumed that 'b' and 'a' are equally likely to occur whereas the probability of occurrence of 'm' is 4 times that of 'b' and 'a'. This is shown in the diagram above.

Thus the Expected Time (t_E) of the activity is given as – $t_E = \frac{1}{6}a + \frac{2}{3}m + \frac{1}{6}b$ Or, $t_E = (a + 4m + b) / 6$

The Expected Time (shown as 't' in the diagram above) divides the area under the curve into two equal parts. Hence it provides the time in which there is a 50 – 50 chance of the activity being completed. This multiple time estimate approach is better since it provides quite often the improved estimates of three expected time to complete an activity and also allows to consider the variability of the time for completion of an activity. A basic reason to estimate more than one time for an activity is to provide data by which the management may determine the probabilities that each activity as well as the entire project will be completed by a specified date. Various formula are used for the purpose along with the table showing area under the Standard Normal Distribution. In this context the important formulae are as follows –

i. Standard Deviation of the time required to complete each activity = (b – a) / 6
ii. Variance of the time required to complete the project = Sum of the Variances of all the activities of the Critical Path From this value Standard Deviation of the time required for the project completion can be easily computed.

Probability of Project completion within a stipulated time

Project completion refers to the occurrence of the end event of the network. So probability of Project completion within a stipulated time means probability of occurrence of the end event of the network within the referred time. This is based on the following –

1. Though it is discussed above that the activity time follows Beta Distribution of Probability, but for the Duration of the complete project it is assumed to follow Normal Distribution. To be precise it is a case of

Normal approximation to Beta Distribution. Thus probability of project completion within a stipulated time can be easily computed using the standard concept of Normal Distribution.

2. Concept of Central Limit Theorem can be used to find out the probability. As per the theorem, when independent random variables are summed up, their properly normalized sum tends towards a Normal Distribution even if the original variables themselves do not follow Normal Distribution.

Procedure of Calculating Probability of Project Completion within a stipulated time (t_S)

(a) Calculate the Mean of the event time by adding the Expected Times of the activities along the Critical Path of the Network.

If 1 – 2 – 4 – 7 – 9 is the Critical Path of the Network then Mean of the Project time (T_E) is given as follows: $T_E = (t_E)_{1-2} + (t_E)_{2-4} + (t_E)_{4-7} + (t_E)_{7-9}$

(b) Calculate Variance of the Project Time $[(\sigma_T)^2]$ by adding Variances of the completion times of the activities along the Critical Path of the Network. Thus $(\sigma_T)^2 = (\sigma_{1-2})^2 + (\sigma_{2-4})^2 + (\sigma_{4-7})^2 + (\sigma_{7-9})^2$. Now calculate S.D ($\sigma_T$) by finding square root of the Variance

[If there is **more than one Critical Path** in the Network then calculate Variance for each of the Critical Path and consider that value of the Variance which is **greatest**]

(c) Find out the value of the Standard Normal variable (Z) corresponding to stipulated time (t_S) by using the formula – $Z = (t_S - T_E) / \sigma_T$

(d) From the Table for Area under the Standard Normal Curve, the value corresponding to the Z value calculated in step (c) gives the required Probability.

Some very commonly used Z values and their corresponding probabilities are given as follows –

$P(Z \leq 1.645) = 95\%$, $P(Z \leq 1.96) = 97.5\%$, $P(Z \leq 2.33) = 99\%$, $P(Z \leq 3) = 99.9\%$ and $P(Z \leq 0) = 50\%$

Illustration 4

A civil engineering firm has to bid for the construction of a dam. The activities and time estimates are given below:

Activity	DURATION		
	Optimistic	Most likely	Pessimistic
1—2	14	17	25
2—3	14	18	21
2—4	13	15	18
2—8	16	19	28
3—4 (dummy)			
3—5	15	18	27
4—6	13	17	21
5—7 (dummy)			
5—9	14	18	20

Activity	DURATION		
	Optimistic	Most likely	Pessimistic
6—7 (dummy)			
6—8 (dummy)			
7—9	16	20	41
8—9	14	16	22

The policy of the firm with respect to submitting bids is to bid the minimum amount that will provide a 95% probability of at best breaking even. The fixed costs for the project are 8 lakhs and the variable costs are 9,000 everyday spent working on the project. The duration is in days and the costs are in terms of rupees.

What amount should the firm bid under this policy? (You may perform the calculations on duration etc. up to two decimal places.)

Solution:

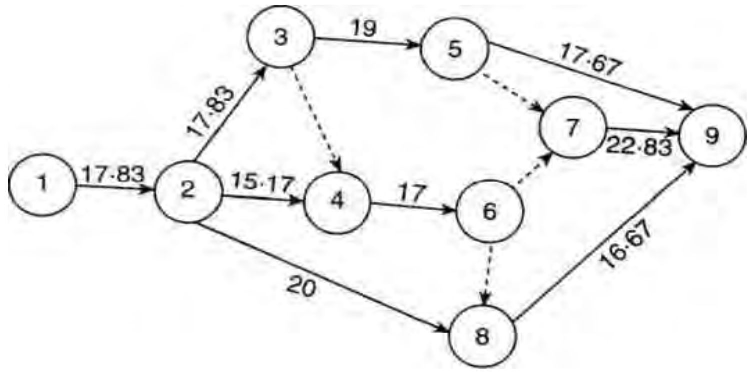
Expected Duration of each activity is calculated using the formula $t_E = (a+4m+b)/6$ and Variance of Duration of each activity is calculated using the formula $\sigma^2 = [(b - a)/6]^2$

where a = Optimistic Duration, m = Most Likely Duration and b = Pessimistic Duration

Calculated values are shown in the Table follows:

Activity	DURATION in Days			Expected duration (t_E Days)	Variance (σ^2 Sq. Days)
	Optimistic (a)	Most likely (m)	Pessimistic (b)		
1—2	14	17	25	17.83	3.36
2—3	14	18	21	17.83	1.36
2—4	13	15	18	15.17	0.69
2—8	16	19	28	20	4
3—4 (dummy)	0	0	0	0	0
3—5	15	18	27	19	4
4—6	13	17	21	17	1.78
5—7 (dummy)	0	0	0	0	0
5—9	14	18	20	17.67	1
6—7 (dummy)	0	0	0	0	0
6—8 (dummy)	0	0	0	0	0
7—9	16	20	41	22.83	17.36
8—9	14	16	22	16.67	1.78

The Network Diagram for the given set of activities is shown below -



Various paths of the Network and their corresponding lengths are as follows –

Sl.	Path	Duration (Days)
I.	1-2-3-5-7-9	77.49*
II.	1-2-3-5-9	72.33
III.	1-2-3-4-6-7-9	75.49
IV.	1-2-3-4-6-8-9	69.33
V.	1-2-8-9	54.50
VI.	1-2-4-6-8-9	66.67
VII.	1-2-4-6-7-9	72.83

From above, the longest path or the path of longest duration of 77.49 Days is 1-2-3-5-7-9. So it is the Critical Path.

Variance of duration = σ^2 = Sum of the Variances of the Critical activities = $(\sigma_{1-2})^2+(\sigma_{2-3})^2+(\sigma_{3-5})^2+\sigma_{5-7})^2+(\sigma_{7-9})^2$ = 3.36+ 1.36+ 4+ 0+ 17.36 = 26.08 Sq. Days

S.D of Project duration = $\sigma = \sqrt{26.08} = 5.11$ Days

To calculate the project duration which will have 95% chance of completion, we find the value of Z corresponding to 95% area under the Standard Normal curve and this value is 1.645.

Therefore P (Project duration $\leq t_s$) = 95% Or, P [$Z \leq (t_s - 77.49)/5.11$] = P ($Z \leq 1.645$)

So, $(t_s - 77.49)/5.11 = 1.645$ Or, $t_s = 86$ Days

Hence the company should bid considering project completion time to be 86 Days.

Amount to bid = Fixed Cost + Variable Cost (@ ₹ 9000 per Day) = ₹ 800000 + ₹ 9000 x 86 = ₹ **15,74,000**

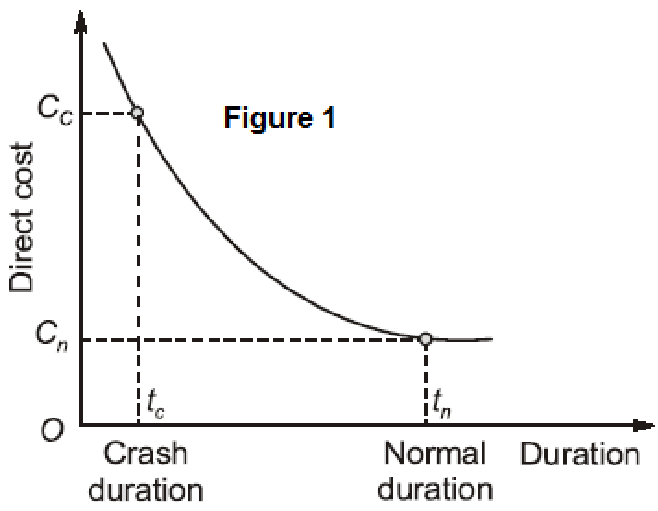
Crashing (Time – Cost Trade Off)

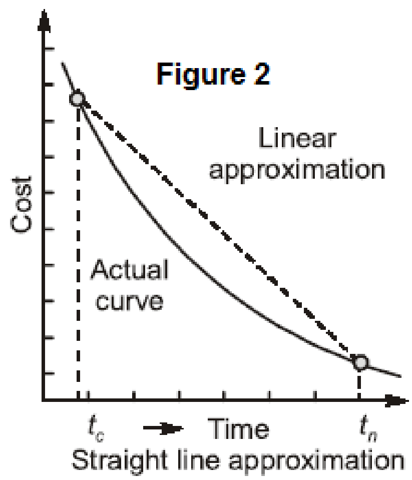
Crashing of a Project Network means intentionally reducing the duration of project by allocating more resources to it. Allocation of more resources apparently means increase in project cost. But this may not be true always and a detail study of relationship of Duration (or Time) and Cost of different activity is necessary to understand it.

Two types of Costs are associated with an Activity – Direct Cost and Indirect Cost.

Direct Costs can be identified with the Activity. For example Cost of Materials, Machinery used in the project and payments made to the Labors and Sub contractors etc. come under this head. So whenever more resources are allocated to an Activity, the Direct Costs are going to increase. Hence Crashing an Activity increases Direct Cost.

Indirect Costs are those which cannot be identified with the Activity. Overheads, Office Expenses, Administrative Costs, Penalty for delay in delivery of project etc. come under this head.





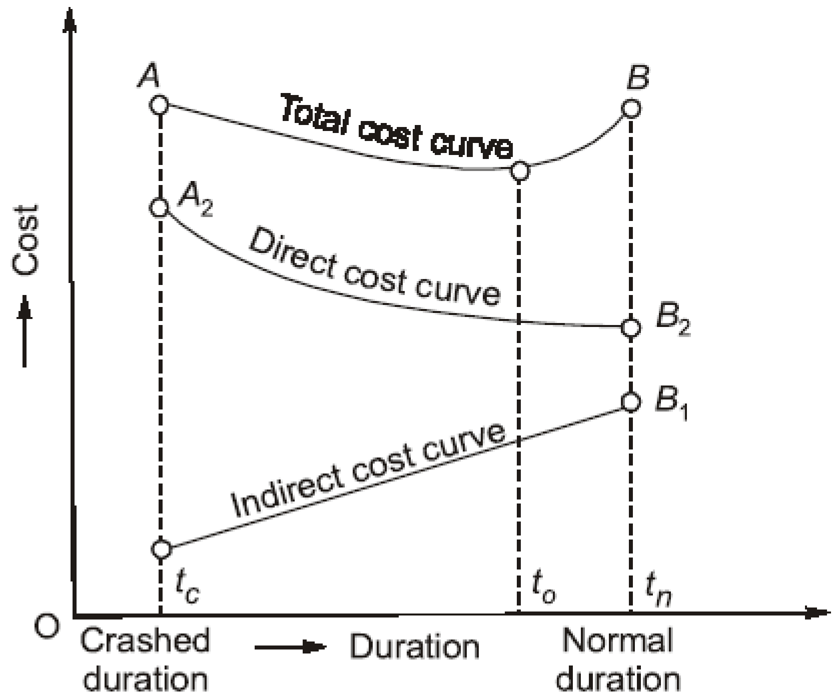
Figures shown above help to visualize the Time - Cost relationship which is sloping downward for at least a limited portion. The actual curve is shown in Figure 1 and it's Linear approximation is shown in Figure 2. The important elements of this curve are

- Normal time (t_n) – It is the standard time an Activity takes for its completion under normal working conditions.
- Crash time (t_C) – It is the minimum time that an Activity requires for its completion.
- Normal cost (C_n) – It is the direct cost required to complete the Activity in Normal Time.
- Crash cost (C_C) – It is the direct cost to be incurred by the Activity for completion in Crash Time.

For keeping the calculations simple, linear approximation of the curve is made and shown in Figure 2. Slope of the straight line is sufficient to define the curve. This slope is known as Cost Slope and given as follows

$$\text{Cost Slope} = \frac{\text{Crash cost} - \text{Normal cost}}{\text{Normal time} - \text{Crash time}}$$

Total Direct Cost of the project can be determined by adding the Direct Costs of each individual activity. When total Indirect Cost is also known then addition of Total Direct and Total Indirect Costs lead to Total Project Cost. A graphical representation of Total Project Cost versus Duration is shown in the diagram below. From this, one thing is clear that at the initial stages of Crashing there is a reduction in Total Project Cost. This happens because rate of decrease in Indirect Cost due to reduction in Project Duration is more than the rate of increase in Direct Cost at this phase. However after a certain point of time there is a reversal of scenario and the Total Project Cost is observed to be increasing again. The duration corresponding to which Total Project Cost is minimum is known as Optimum Duration of the Project. Crashing is always done up to this optimum point. In the diagram below t_o is the Optimum Duration of the Project.



Procedure of Crashing a Project Network

Crashing is an activity that refers to taking specific costly measures to decrease the duration of an activity. These specific measures may involve Usage of Overtime, Outsourcing, Usage of special Equipment etc. Crashing the project refers to crashing a number of activities which leads to decrease the duration of the project less than its normal value.

Procedure of Crashing a Project Network ia given as follows –

- Draw the Network diagram with the Normal Activity Times.
- Identify the Critical Path.
- Compute Cost Slope of different critical activities using the formula written above.
- Rank the activities of the Critical Path in ascending order of Cost Slope.
- Crash the activities in the Critical Path following the rule – "Lowest one first" i.e Critical Activity having lowest ranking should be crashed first to the maximum extent possible. The Activity must not be crashed to

that extent which will make the Critical Path non critical. At the most along with the existing Critical Path some other path can be allowed to become critical. Calculate new Total Cost by adding Cost of Crashing to the cumulative Normal Cost.

- Once more than one path becomes Critical, parallel crashing of activities in all the critical paths will become necessary.
- Repeat the above step of Crashing until no further crashing is possible or no further reduction in Total Cost is possible.

Illustration 5

The following table gives data on normal time & cost as well as crash time & cost for a project.

You need to draw the Network diagram and identify the Critical Path.

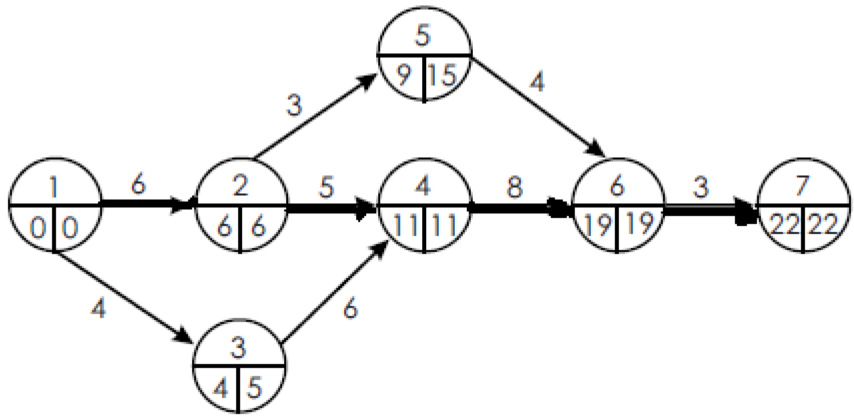
Also find out the Normal duration of the project and the corresponding Total Cost associated with it.

Crash the relevant activities systematically and determine the optimum completion time of the project. Also determine the corresponding cost when it is given that the Indirect Cost is ₹ 100 per day.

Activity	Normal		Crash	
	Time (days)	Cost (₹)	Time (days)	Cost (₹)
1—2	6	600	4	1,000
1—3	4	600	2	2,000
2—4	5	500	3	1,500
2—5	3	450	1	650
3—4	6	900	4	2,000
4—6	8	800	4	3,000
5—6	4	400	2	1,000
6—7	3	450	2	800

Solution:

The network for normal activity times indicates project duration of 22 days with critical path 1-2-4-6-7. It is shown below



Total Cost associated with it is given as (Normal Direct Cost + Indirect Cost for 22 Days @ ₹ 100 per Day)

Normal Direct Cost = (600 + 600 + 500 + 450 + 900 + 800 + 400 + 450) = ₹ 4700

Indirect Cost = 22 × 100 = ₹ 2200

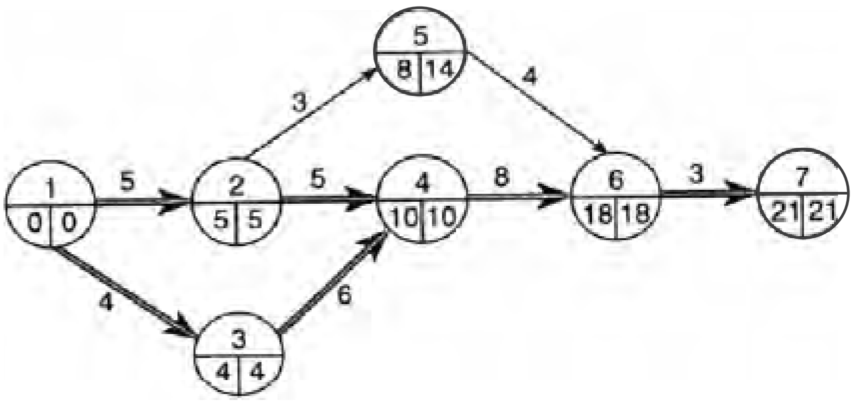
Required Total Cost = 4700 + 2200 = ₹ 6900

1st Stage of Crashing

Cost slope of each of the Critical Activities of the Network diagram is calculated and ranked as below.

Critical Activity	Cost Slope =	Rank as per ascending order of Cost Slope
1 - 2	(1000 – 600)/ (6 – 4) = ₹ 200 per day	1
2 – 4	(1500 – 500)/ (5 – 3) = ₹ 500 per day	3
4 – 6	(3000 – 800)/ (8 – 4) = ₹ 550 per day	4
6 – 7	(800 – 450)/ (3 – 2) = ₹ 350 per day	2

As Cost Slope of Activity 1 – 2 is minimum, crashing is to be started from this Activity. Maintaining criticality of the existing Critical Path, Activity 1 – 2 is crashed by 1 Day.



New Network Diagram is shown above. It is having Duration of 21 Days and the associated Total Cost is given as TC = Normal Direct Cost + Indirect Cost (for 21 Days @ ₹ 100 per Day) + Cost of Crashing Activity 1-2 by 1 Day

= 4700 + 21 × 100 + 1 × 200 = ₹ 7000

It is seen that other activities too have become Critical. Now there are two Critical Paths given by 1 – 2 – 4 – 6 – 7 as well as 1 – 3 – 4 – 6 – 7

2nd Stage of Crashing

Cost Slopes of each of the new Critical Activities are calculated as below.

Cost Slope of Activity 1 – 3 = (2000 – 600)/ (4 – 2) = ₹ 700 per Day & that of 3 – 4 = (2000 – 900)/ (6-4) = ₹ 550/- per Day.

As there are more than one Critical Path, parallel Crashing is necessary for some of the activities to maintain criticality of the existing Critical Paths. Various options of Crashing and their corresponding Cost Slopes are shown below.

Options	Possible Crash (Days)	Cost Slope (₹/ Day)	Rank
Activities (1 - 2) & (1 - 3)	1*	200 + 700 = 900	4
Activities (1 - 2) & (3 - 4)	1*	200 + 550 = 750	3
Activities (2 - 4) & (1 - 3)	2	500 + 700 = 1200	6
Activities (2 - 4) & (3 - 4)	2	500 + 550 = 1050	5
Activity (4 – 6)	4	550	2
Activity (6 – 7)	1	350	1

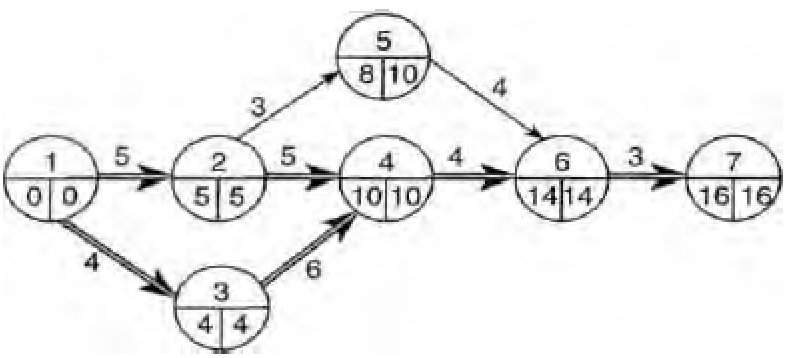
* Though as per the supplied data activities (1-3) & (3-4) can be crashed by 2 days each, but (1 – 2) cannot be crashed more than 1 Day after 1st stage of Crashing.

From the above ranking Crashing of (6-7) by 1 Day is suggested. Due to this project duration will be 20 Days and associated Total Cost = Normal Direct Cost + Indirect Cost for 20 Days @ ₹ 100 per Day+ Crashing Cost of Activity (1 – 2) by 1 Day @ ₹ 200 per Day + Crashing Cost of Activity (6 – 7) by 1 Day @ ₹ 350 per Day = 4700 + 20 × 100 + 1 × 200 + 1 × 350 = ₹ 7250

3rd Stage of Crashing

After 2nd Stage of Crashing, no new Critical Path emerged. So the options remain same as in the 2nd Stage with the exception of Activity (6 – 7) which is totally crashed in the 2nd Stage.

From the above list of Ranking, Activity (4 – 6) is having lowest Cost Slope. Thus it is crashed by 4 days now. New Network having project duration of 16 Days is shown below.

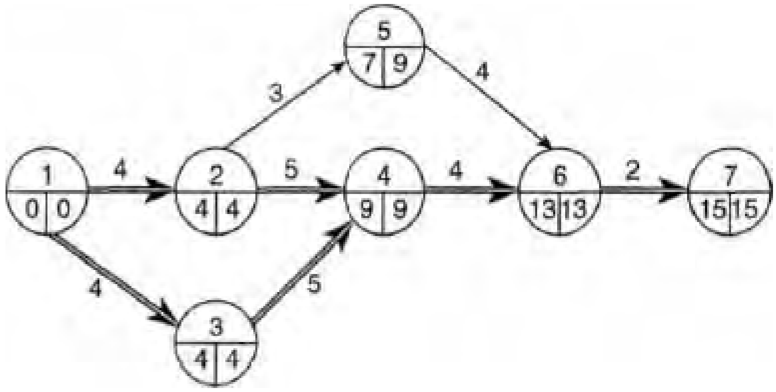


Total Cost of the Project = Normal Direct Cost + Indirect Cost (for 16 Days @ ₹ 100/ Day) + Crashing Cost [for Activity (1 – 2) by 1 Day @ ₹ 200/ Day + for Activity (6 – 7) by 1 Day @ ₹ 350/ Day + for Activity (4 – 6) by 4 Days @ ₹ 550/ Day] = 4700 + 1600 + 200 + 350 + 550 × 4 = ₹ 9050

4th Stage of Crashing

After 3rd Stage of Crashing, no new Critical Path emerged. So the options remain same as in the 2nd Stage with the exception of Activities (6 – 7) and (4 – 6) which are fully crashed in the 2nd and 3rd Stages.

From the above list of Ranking, Activity (1 – 2) and (3 – 4) together is having lowest Cost Slope. Thus both are crashed by 1 day now. New Network having project duration of 15 Days is shown below.



Total Cost of the Project

= Normal Direct Cost + Indirect Cost (for 15 Days @ ₹ 100/ Day) + Crashing Cost [for Activity (1 – 2) by 1 Day @ ₹ 200/ Day + for Activity (6 – 7) by 1 Day @ ₹ 350/ Day + for Activity (4 – 6) by 4 Days @ ₹ 550 per Day + for Activities (1 – 2) & (3 – 4) together by 1 Day @ ₹ 750/Day]

= 4700 + 1500 + 200 + 350 + 550 × 4 + 750 = ₹ 9700

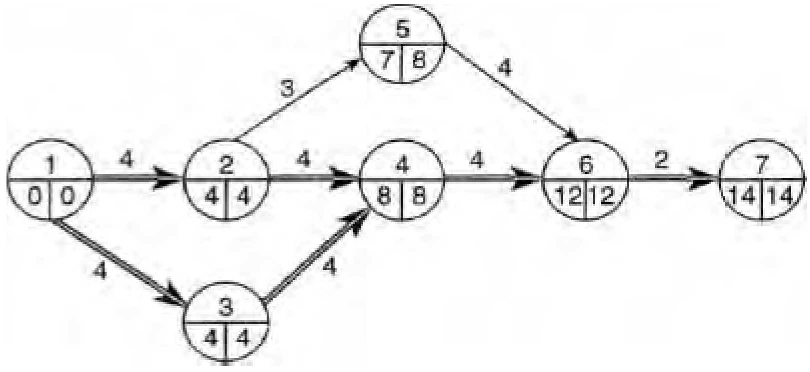
5th Stage of Crashing

Though after 4th Stage of Crashing no new Critical Paths emerged, but the Activity (1 – 2) has been crashed fully. Thus the options remaining are as follows.

Options	Possible Crash (Days)	Cost Slope (₹/ Day)	Rank
Activities (2 - 4) & (1 - 3)	2	500 + 700 = 1200	2
Activities (2 - 4) & (3 - 4)	1*	500 + 550 = 1050	1

* Though Activity (2 - 4) can be crashed by 2 Days but after 4th Stage, (3 – 4) has only 1 Day of Crashing left.

As Cost Slope of Activities (2 – 4) & (3 – 4) taken together is least, both are crashed by 1 Day and the new Network diagram is shown below. It shows project duration of 14 Days.



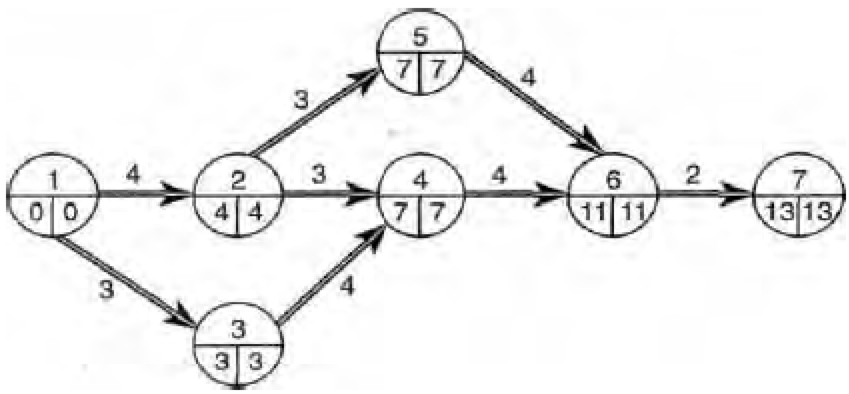
Total Cost of the Project

= Normal Direct Cost + Indirect Cost (for 14 Days @ ₹ 100/ Day) + Crashing Cost [for Activity (1 – 2) by 1 Day @ ₹ 200/ Day + for Activity (6 – 7) by 1 Day @ ₹ 350/ Day + for Activity (4 – 6) by 4 Days @ ₹ 550/ Day + for Activities (1 – 2) & (3 – 4) together by 1 Day @ ₹ 750/Day + for Activities (2 – 4) & (3 – 4) together by 1 Day @ ₹ 1050/ Day]

= 4700 + 1400 + 200 + 350 + 550 X 4 + 750 + 1050 = ₹ 10650

6th Stage of Crashing

After 5th Stage of Crashing no new Critical Paths emerged. So the available option as per the table above is to crash (2 – 4) and (1 – 3) together and they can be crashed by 1 Day because after 5th Stage only 1 Day of crashing is available for Activity (2 – 4). The new Network diagram having project duration of 13 Days is shown below.



Total Cost of the Project

= Normal Direct Cost + Indirect Cost (for 13 Days @ ₹ 100/ Day) + Crashing Cost [for Activity (1 – 2) by 1 Day @ ₹ 200/ Day + for Activity (6 – 7) by 1 Day @ ₹ 350/ Day + for Activity (4 – 6) by 4 Days @ ₹ 550 per Day + for Activities (1 – 2) & (3 – 4) together by 1 Day @ ₹ 750/Day + for Activities (2 – 4) & (3 – 4) together by 1 Day @ ₹ 1050/ Day + for Activities + for Activities (2 – 4) & (1 – 3) by 1 Day @ ₹ 1200/ Day]

= 4700 + 1300 + 200 + 350 + 550 X 4 + 750 + 1050 + 1200 = ₹ 11750

From the diagram it is clear that all the paths of the Network are Critical. Also activities of the path 1 – 2 – 4 – 6- 7 are each fully crashed. Thus no further crashing of the Network is possible.

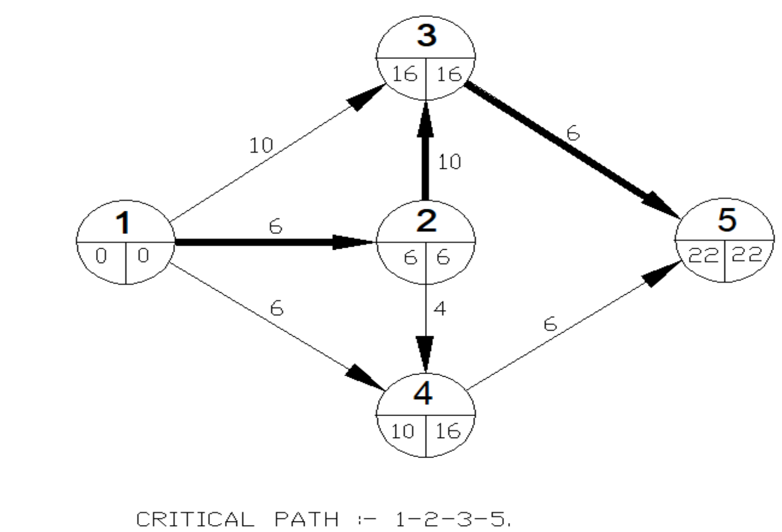
It is noticed that the Total Cost of the Project kept on increasing all along. This has happened due to the fact that the rate of decrease of Indirect Cost is much lower than the rate of increase of Direct Cost for Crashing. Hence optimum duration of the project cannot be obtained and rather minimum possible duration is obtained and that value is 13 Days. Associated Total Cost of project is ₹ 11750.

Illustration 6 (Resource levelling)

Consider the following problem of project scheduling to obtain a schedule which will minimize the peak manpower requirement and smooth out period to period variation of manpower requirement.

Activity	Duration (Days)	Manpower requirement
1-2	6	8
1-3	10	4
1-4	6	9
2-3	10	7
2-4	4	6
3-5	6	17
4-5	6	6

Solution:

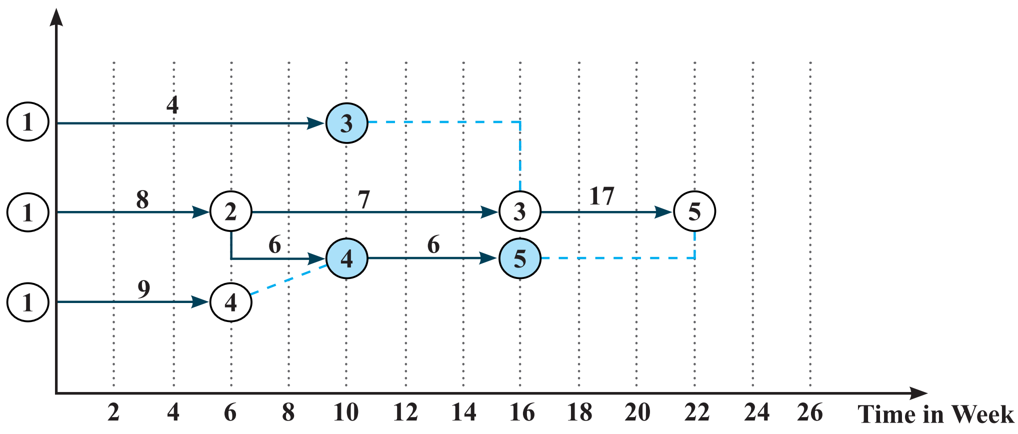


The earliest & Latest Times for each event are presented in the Nodes.

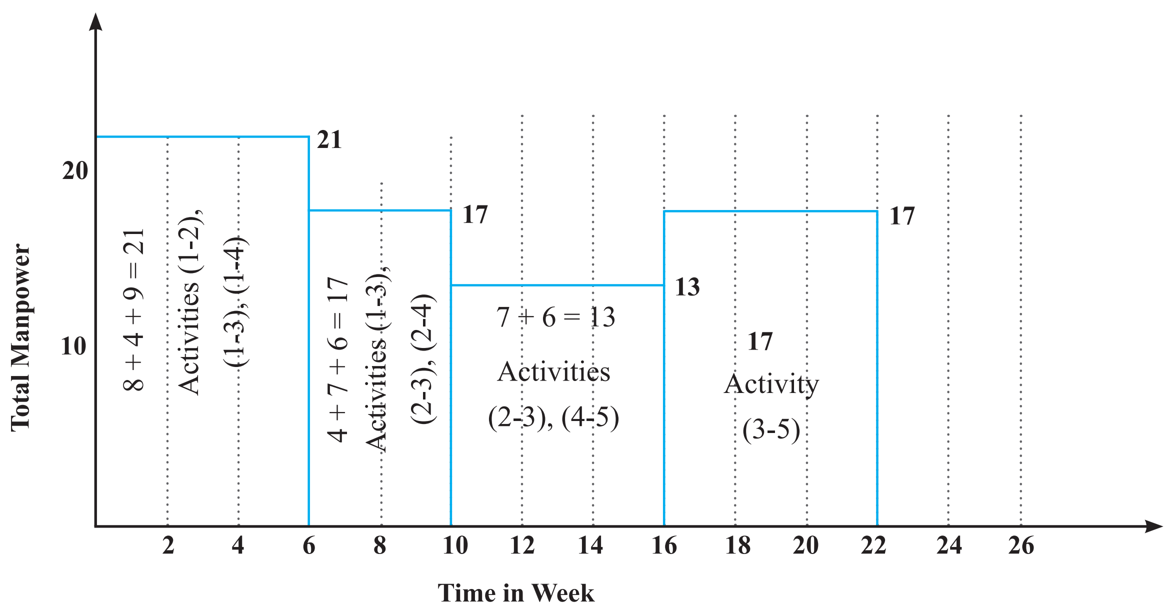
The Network Diagram is shown in the figure above. Critical path = 1-2-3-5

The Earliest & Latest Times for each event are presented in quadrants of the Node Circles.

Now the activities are represented on a time scale as shown below. The corresponding Manpower requirement are mentioned above the arrows representing Activities in the diagram.

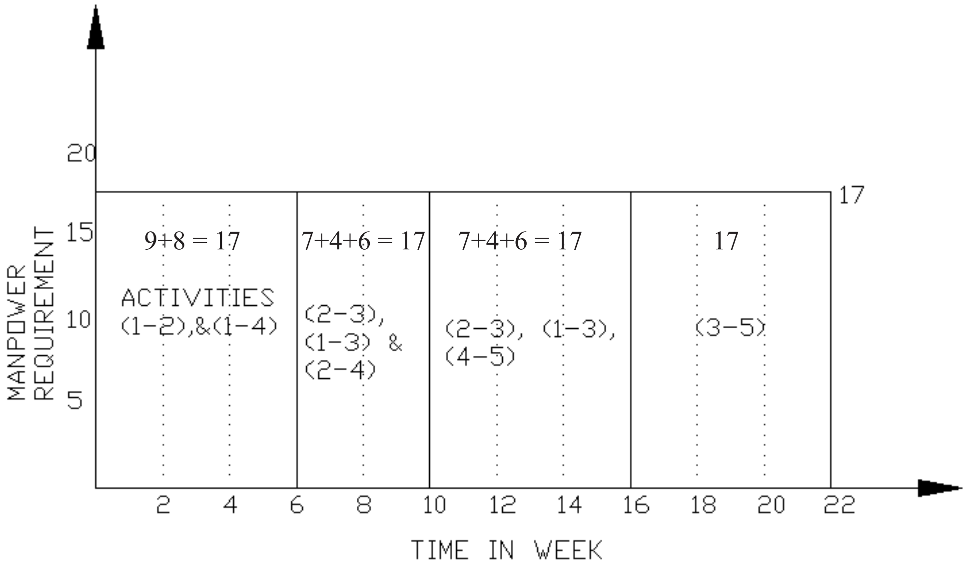


The corresponding Manpower Requirement Histogram (shifting activity 4 – 5 to the right) is as follows.



The peak manpower requirement is 21 and it occurs between 0-6 weeks. The activities which are scheduled during this period are (1 – 2), (1 – 3) and (1 – 4). Activity (1 – 2) is a Critical Activity. So it should not be disturbed. Between activities (1 – 3) and (1 – 4), the activity (1 – 3) has a Total Float of 6 weeks. So it has to be delayed to the maximum possible extent (i.e it can be started at the end of the 6th week).

The modified Histogram is shown below.



The manpower requirement is now balanced / smoothened throughout the project duration.

EXERCISE

A. Theoretical Questions:

Multiple Choice Questions

1. Critical Activities have

   (a) Maximum float

   (b) Minimum float

   (c) Zero float

   (d) Negative float

2. In PERT Chart, the Activity time distribution is -

   (a) Normal

   (b) Binomial

   (c) Poisson

   (d) Beta

3. A PERT Network has nine activities on its Critical Path. The Standard Deviation of each activity on the Critical Path is 3. The S. D of the Critical Path is –

   (a) 3

   (b) 9

   (c) 81

   (d) 27

4. For an activity the pessimistic, most likely and optimistic times are respectively 10, 6 and 2 days. The expected duration of the activity is –

   (a) 6 days

   (b) 3 days

   (c) 2 days

   (d) 9 days

5. The time by which the activity completion time can be delayed without affecting the start of the succeeding activities is known as –

   (a) Total float

   (b) Free float

   (c) Independent float

   (d) Head slack

6. Which of the following statement is not true?

   (a) PERT is deterministic in nature.

   (b) CPM is probabilistic in nature.

(c) PERT Network can not be crashed.

(d) All of the above.

7. Following data refers to a project Network. What will be the Critical Path?

Activity	1 – 2	2 – 3	3 – 4	1 – 4	2 – 5	3 – 5	4 – 5
Duration	2 Days	1 Day	3 Days	3 Days	3 Days	2 Days	4 Days

(a) 1 – 2 – 3 - 5

(b) 1 – 2 – 3 – 4 – 5

(c) 1 – 4 – 5

(d) 1 – 4 – 3 – 5

8. The amount of time by which an activity can be delayed without affecting the project completion is called –

(a) Free float

(b) Total float

(c) Interfering float

(d) None of the above

9. Optimistic time and pessimistic time of an activity are respectively 4 days and 16 days. Variance of the duration of the activity will be –

(a) 4 day2

(b) 2 day2

(c) 3 day2

(d) None of the above

10. In a project planning Free float can affect which of the following?

(a) Succeeding activity

(b) Only that activity

(c) Preceding activity

(d) All of the above

11. Solution of problems of Crashing has to be started by applying the technique on –

(a) Any activity of the Network.

(b) Non critical activities.

(c) Critical activities.

(d) None of the above.

12. A PERT activity has an optimistic time of 3 days, pessimistic time of 15 days and an expected time of 7 days. What is the most likely time of the activity?

(a) 10 days

(b) 6 days

(c) 5 days

(d) None of the above

13. The reduction in project time normally results in –

(a) Decrease in Direct Cost and increase in Indirect Cost

(b) Increase in Direct Cost and decrease in Indirect Cost

(c) Increase in both Direct and Indirect Costs.

(d) Decrease in both Direct and Indirect Costs.

14. The Normal duration and Normal cost of an activity are respectively 10 days and ₹ 350. The cost slope is ₹ 75 per day. If the Crash duration is 8 days then what is the Crash cost of the activity?

(a) ₹ 400/-

(b) ₹ 500/-

(c) ₹ 600/-

(d) ₹ 650/-

15. Which of the following is incorrect?

(a) PERT is suitable for projects having probabilistic time estimates.

(b) CPM is suitable for projects having deterministic activities.

(c) Both PERT and CPM are event oriented.

(d) PERT is event oriented while CPM is activity oriented.

16. The activity that must be completed prior to the start of an activity is called –

(a) Dummy activity

(b) Successor activity

(c) Concurrent activity

(d) Predecessor activity

17. The slack times of Tail and Head events of Activity P are respectively 10 days and 4 days. If the Free float of the Activity P is 12 days then the Total float would be –

(a) 8 days

(b) 16 days

(c) 22 days

(d) None of the above

18. Which of the following represents reduction in project duration?

(a) Crashing

(b) Negative slack

(c) Variance

(d) All of the above

19. Critical Path Method is good for –
    (a) Small projects only
    (b) Large projects only
    (c) Both small and large projects equally
    (d) Neither small nor large projects

20. The optimum duration is the –
    (a) Summation of normal durations of each activity of the project.
    (b) Summation of normal durations of activities in the Critical Path.
    (c) One which gives the minimum Total Cost for the completion of the project.
    (d) Summation of crash durations of activities in the Critical Path.

21. Which of the following is not a notable challenge while scheduling a project?
    (a) Deadlines exist
    (b) Independent activities
    (c) Too many workers may be required
    (d) Costly delay

22. A critical path is –
    (a) The shortest path
    (b) The longest path
    (c) The path that begins from the start node and ends at the last node.
    (d) All of the above.

23. Activities A, D and F merges at the event 6. If the earliest finish times of A, D and F are respectively 13, 17 and 8 then the earliest time of Event 6 is –
    (a) 8
    (b) 13
    (c) 17
    (d) Cannot be determined from the given information.

24. Which of the following is true when a project is scheduled by Critical Path Analysis?
    (a) Work breakdown structure is used to divide the project into different activities.
    (b) Duration for each activity is established.
    (c) Precedence relationship of the activities is determined.
    (d) All of the above.

25. Total Project Cost versus Duration curve is –
    (a) Parabolic
    (b) S shaped

(c) U shaped

(d) Linear

26. Activities P, Q and R are the immediate successors of the activity N. If their current starting times are 10, 11 and 17 respectively then what is the latest finishing time of the activity N?

(a) 10

(b) 11

(c) 17

(d) None of the above

27. Activity in a Network diagram is represented by –

(a) Circle

(b) Rectangle

(c) Square

(d) Arrow

28. The particular task performance in CPM is known as –

(a) Event

(b) Activity

(c) Dummy

(d) Contract

29. Among the following, critical path and slack time analysis mostly help

(a) Managers define the project activities

(b) Highlight relationships among project activities.

(c) Point out who is responsible for various activities

(d) Pinpoint activities that need to be closely watched.

Answers:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
c	d	b	a	b	d	b	b	a	a	c	b	b	b	c
16	17	18	19	20	21	22	23	24	25	26	27	28	29	
d	b	a	b	c	b	b	c	d	c	a	d	b	d	

State True or False

1. Full form of PERT is Program Evaluation and Robot Technology.

2. The difference between the maximum time available and the actual time needed to perform an Activity is called the Total Float of the Activity.

3. PERT Network is Activity oriented and CPM Network is Event oriented.

4. An activity can have three different types of floats.
5. PERT and CPM are both project evaluation techniques.
6. The objective of Network Analysis is to minimize total project duration.
7. For any activity, Independent Float ≤ Free Float ≤ Total Float
8. Events neither consume any resource nor any time.
9. P (Z ≤ 2.33) = 95%, is a very important result used for finding the duration of a project which has 95% chance of completion within time.
10. In a CPM Network diagram completion of an activity is known as Event or Node or Connector
11. Project crashing is possible for both CPM and PERT.
12. As per Backward Pass technique, Latest allowable Event Time is $L_i = L_j - t_{ij}$ & for Burst Events $L_i =$ Min. $(L_j - t_{ij})$
13. In a Network diagram Activities are represented by arrows having special importance on the lengths of the same according to the duration of the activity.
14. In a Construction project, "Curing of Concrete" is an Activity but "Concrete Cured" is an Event.
15. Critical Path of a Network helps the Project Manager to divert the resources from non-critical advanced activities to critical activities.

Answers:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
F	T	F	T	F	T	T	T	F	T	F	T	F	T	T

- **Fill in the blanks**

1. The activity with minimum _____ should be crashed first.
2. ____ Pass technique is used to get the Earliest Event times.
3. For any Merge Event, earliest time is obtained by taking ____ value of the Earliest Finish times of all the activities ending there.
4. Drawing unnecessary dummy activity in a Network diagram is called _____ error.
5. Simultaneous accomplishment of those activities in a Network is possible which are ______.
6. Slack of the event at the arrowhead end of an activity is known as _____.
7. In a PERT Activity, probability of Most Likely Time is ____ times of that of Optimistic Time.
8. Standard rule of Network drawing says that the value of i is ____ that of j always, for the Activity (i – j).
9. For R & D type Projects, ______ is the most suitable technique of drawing Network diagrams

10. Dummy activities are drawn to make activities with ____ beginning and ending events distinguishable.
11. An Event where more than one activity ends and also from where more than one activity begins is called _____ Event.
12. In a PERT Network, probability of completion of a project within a stipulated time is estimated by using ____-approximation to β Distribution.
13. Network is a technique, used for Planning, Scheduling and _____ of any project.
14. The value of Slack of all the events in the Critical Path of a Network is ____.
15. Sum of the Free float and Head slack gives _____ float of an activity.

Answers:

1.	Cost slope	**2.**	Forward
3.	Maximum	**4.**	Redundancy
5.	Concurrent	**6.**	Head Slack
7.	Four	**8.**	Less than
9.	PERT	**10.**	Common
11.	Merge and Burst	**12.**	Normal
13.	Controlling	**14.**	Zero
15.	Total		

Short essay type questions

1. What is a Path in a Network diagram?
2. What is Crash Duration of an Activity?
3. Write short note on Expected Duration of a PERT Activity?
4. What is the difference between AOA Diagram and AON Diagram in Network?
5. What is Looping error in Network Diagram?
6. Define Cost Slope in relation to Crashing of a project Network.
7. Write short notes on Event and Merge Event in a Network.

Essay type questions

1. Distinguish between PERT and CPM.
2. Explain the following terms in PERT – (a) Optimistic Time, (b) Pessimistic Time, (c) Most Likely Time, (d) Variance of duration of Activity, (e) Project duration Variance, (f) Mean completion time of the project.
3. Write short notes on – Total Float, Free Float, Independent Float, Head Slack and Tail Slack.
4. Explain the meaning of Crashing in Network techniques.

5. What is Network Analysis and when it is used? What is meant by Critical Path?

B. Numerical Questions

Comprehensive Numerical Problems

1. Information about a project are given as follows

Activity	A	B	C	D	E	F	G	H	I	J	K
Predecessor Activity	None	None	A	None	B, C	D	D	E, F	E, F	G, I	H
Least time	4	5	8	2	4	6	8	5	3	5	6
Greatest time	8	10	12	7	10	15	16	9	7	11	13
Most likely time	5	7	11	3	7	9	12	6	5	8	9

Draw the Network diagram and identify the Critical Path. Find the Earliest and Latest times of each Event. Also calculate the Total Float for each activity. Consider the unit of various times given above is weeks.

2. [**Case Study**] M/S Ramswarup Limited is a renowned trader of Steel Structural with head office in Kolkata, has planned to diversify and decided to enter into manufacturing sector. They want to put up a Foundry with an aim to produce high quality abrasion resistant castings which has a very good export potential. A detailed study has been carried out by the team of M/S Ramswarup with the help of consultant Dr. P.K.Mathur who is a management consultant with vast hands on experience in manufacturing of castings. After the study, Dr. Mathur suggested not to go for a huge installed capacity at the beginning. Rather he is of the opinion to go in phases. Accordingly he listed the following necessary activities with associated precedence relationships as well as Expected Durations and S.Ds in weeks for the first phase operation.

Activity	A	B	C	D	E	F	G	H	I	J
Predecessor	None	None	A	A	A	C	D	B, D, E	H	I, G, F
Duration	6	3	5	4	3	3	5	5	2	3
S.D	1	0.5	1	1	0.5	0.5	1	1	0.5	1

You are entrusted with the responsibility of assisting Dr. Mathur in different areas as he deemed necessary. To properly Plan, Schedule and Control the first phase of the project, Dr. Mathur wants you to draw the Network diagram and identify the Critical Path. Also he wants to know the project duration.

After getting the Network diagram from you, Dr. Mathur submitted the same to the top management of the company for getting a green signal to start the work. Till the permission is obtained, Dr. Mathur envisaged some delay in the Activities B, E and G by 3, 2 and 2 weeks respectively. He wants to know from you the effect of these delays on the project completion. It has to be reported to him urgently so that he can subsequently bring the same to the notice of the management.

The durations mentioned in the above table are expected values which are Normally distributed with the Standard Deviations (S.D) mentioned above. Ignoring the delays mentioned above for the activities B, E and G and possible effect of uncertainty in the non - critical activities, determine 95% Confidence Interval for the Expected Project Completion.

The Cost of the project is estimated to be ₹ 10,00,000/-. If it is completed within 24 weeks the expected returns should be ₹10,000,000/-. But if the deadline fails the export market will be penetrated by competitors from South Africa and Australia. That will result in a net revenue of only ₹ 2,00,000/-. You need to determine the expected returns ignoring the delays mentioned before and report it to Dr. Mathur.

[**Hints** – Critical Path is A – D - Dummy – H – I – J and project duration is 20 weeks (Can be obtained by drawing the Network diagram)

On calculation of Total Float, the values of the same for Activities B, E and G are respectively 7, 1 and 2 weeks. So delay of 3 weeks in B and 2 weeks in G are not going to hamper the project completion. But delay of 2 weeks in E is going to increase the project duration by 1 week because it is having a Total Float of 1 week.

Mean project duration = 20 weeks (from Network diagram)

Variance of project duration = Sum of the variances of the Critical Activities i.e A, D, H, I & J = $1^2 + 1^2 + 1^2 + (0.5)^2 + 1^2$

S.D of project duration = $\sqrt{[1^2 + 1^2 + 1^2 + (0.5)^2 + 1^2]}$ = 2.06 weeks

So 95% Confidence Interval of the expected project completion = Mean duration ± 1.96 (S.D) = 20 ± 1.96 × 2.06 = 15.96 to 24.04 weeks

P (Project duration exceeds 24 weeks) = P (X > 24) = P [Z > (24 – 20)/ 2.06] = P [Z > 1.94) = 1 – P (Z ≤ 1.94) = 1 – 0.9738 = 0.0262

Expected return from the project = 10,000,000 × P (Project will be completed within 24 weeks) + 2,00,000 × P (Project will not be completed within 24 weeks) – Initial investment = 10, 000, 000 × (1 – 0.0262) + 2,00, 000 × 0.0262 – 10, 00, 000 = ₹ 87, 43, 240/-]

3. The following table gives the activities and other relevant data for a project

Activity	1 – 2	1 – 3	1 – 4	2 – 3	2 – 5	3 – 5	4 - 5
Normal Time (Days)	4	2	5	7	7	2	5
Crash Time (Days)	3	2	4	5	6	1	4
Normal Cost (₹)	600	400	750	400	800	500	600
Crash Cost (₹)	800	400	900	600	1000	650	850

Indirect cost for the project is ₹ 200 per day.

Draw the Network diagram. Find Normal duration and cost of the project. Also find optimum duration and cost for it.

4. Activity P is followed by Activity Q which in turn, is followed by Activity R. The Direct Cost of these Activities in relation to the choice of feasible durations is given in the Table below. For all the three activities taken together what is the minimum possible Direct Cost for a total duration of 21 days.

	Activity P			Activity Q			Activity R		
Duration (Days)	7	6	5	8	7	6	9	8	7
Direct Cost (₹ '000)	12	14	15	20	23	27	40	42	45

5. A project has four activities P, Q, R and S as shown below.

Activity	Normal Duration (Days)	Predecessor Activity	Cost Slope (₹ / DAY)
P	3	-	500
Q	7	P	100
R	4	P	400
S	5	R	200

Normal Cost of the project is ₹ 10000/- and the Overhead Cost is ₹ 200 per day. If the project duration has to be crashed down to 9 days, then what will be the Total Cost of the project?

Answers:

1. Critical Path: A – C – E – H – K,

   Values of Total Floats (in weeks) for the Activities A through K are respectively 0, 8.83, 0, 10, 0, 10, 15, 0, 2.5, 2.5, 0

2. Refer to the Hints given with the question.

3. Normal project duration = 13 days and cost = ₹ 6650/-, Optimum project duration = 10 days and cost = ₹ 6450

4. ₹ 77000

5. ₹ 12500

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